Pensieve header: Evaluating $\rho 1$ and $\rho 2$ at $T=1$.

In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2020-01"];
<< KnotTheory`
$k = 2;
```

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification.

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ToFileName: String or list of strings expected at position 1 in ToFileName[{File, WikiLink, mathematica}].

ToFileName: String or list of strings expected at position 1 in ToFileName[{File, QuantumGroups}].

```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[ ]:=
```
𝔼[L_, Q_, P_]$k_ := 𝔼[L, Q, Series[Normal@P, {ϵ, 0, $k}]];
𝔼d_→r_[L_, Q_, P_]$k_ := 𝔼d→r @@ 𝔼[L, Q, P]$k;
```

In[ ]:=
```
𝔼3@𝔼[ω_, L_, Q_, Ps_] := Simplify /@ 𝔼[L, ω^-1 Q, ω^-1 (ω^-4 ϵ)^(-1+Range@Length@Ps .Ps)]$k;
𝔼3@𝔼sp___[as___] := 𝔼3@𝔼[as] /. 𝔼 → 𝔼sp;
```

In[ ]:=
```
Clear[QP, ω];
QP[Knot[n_, k_]] := QP[Knot[n, k]] = Module[{fname},
    fname =
     "../Projects/SL2Invariant/k=2/Data/" <> ToString[n] <> "_" <> ToString[k] <> ".m";
    Collect[𝔼3[Get[fname]⟦2, 2⟧]⟦3⟧ // Normal, ϵ, Simplify]
   ];
ω[K_Knot] := ω[K] = Factor[(QP@K /. ϵ → 0)^-1];
ck_,d_[K_Knot] :=
 Factor[SeriesCoefficient[QP[K], {y, 0, 0}, {ϵ, 0, k}, {a, 0, d}] ω[K]^(1+2 k-d)]
```

In[ ]:=
```
p1[K_Knot] := p1[K] = Factor[T (-c1,0[K] + ω[K] T ∂T ω[K]) / (T - 1)^2];
```

In[ ]:=
```
p2[K_Knot] := p2[K] = Expand[-2 c2,0[K] + ω[K] c2,1[K]];
```

```
In[•]:= MatrixForm@(mat =
          Table[{1, Vassiliev[2][K], Vassiliev[3][K], p1[K] /. T → 1}, {K, AllKnots[{3, 7}]}])
```

Get: ParentDirectory[File] in $Path is not a string.

KnotTheory: Loading precomputed data in PD4Knots`.

Get: ParentDirectory[File] in $Path is not a string.

KnotTheory: Loading precomputed data in Jones4Knots`.

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & -1 & 0 & 0 \\ 1 & 3 & -5 & 10 \\ 1 & 2 & -3 & 6 \\ 1 & -2 & 1 & -2 \\ 1 & -1 & 1 & -2 \\ 1 & 1 & 0 & 0 \\ 1 & 6 & -14 & 28 \\ 1 & 3 & -6 & 12 \\ 1 & 5 & 11 & -22 \\ 1 & 4 & 8 & -16 \\ 1 & 4 & -8 & 16 \\ 1 & 1 & -2 & 4 \\ 1 & -1 & -1 & 2 \end{pmatrix}$$

```
In[•]:= NullSpace[mat]
Out[•]= {{0, 0, 2, 1}}

In[•]:= Vassiliev[4, 1][K_] := Coefficient[Conway[K][z], z^4]

In[•]:= Table[Vassiliev[4, 1][K], {K, AllKnots[{3, 7}]}]
Out[•]= {0, 0, 1, 0, 0, -1, 1, 5, 0, 2, 0, 2, -1, 1}

In[•]:= Vassiliev[4, 2][K_] := SeriesCoefficient[4 Jones[K][e^x], {x, 0, 4}]

In[•]:= mat = Table[{Vassiliev[4, 1][K], Vassiliev[4, 2][K]}, {K, AllKnots[{3, 8}]}]
Out[•]= {{0, -29}, {0, 5}, {1, -243}, {0, -130}, {0, 34}, {-1, 41}, {1, -17}, {5, -954}, {0, -351},
  {2, -697}, {0, -452}, {2, -476}, {-1, -89}, {1, 17}, {0, 135}, {-3, 108}, {0, 68}, {-2, 63},
  {-3, 209}, {-2, 154}, {3, -94}, {2, -58}, {-3, 46}, {3, -123}, {-2, 125}, {1, 51},
  {2, -53}, {-2, 24}, {3, -320}, {2, -53}, {-2, 29}, {-1, 7}, {5, -565}, {1, -70}, {-1, 84}}

In[•]:= MatrixRank[mat]
Out[•]= 2
```

In[•]:=
```
MatrixForm@
 (mat = Table[{1, Vassiliev[2][K], Vassiliev[3][K], Vassiliev[2][K]^2, Vassiliev[4, 1][K],
    Vassiliev[4, 2][K], p1[K] /. T → 1, p2[K] /. T → 1}, {K, AllKnots[{3, 8}]}])
```

Out[•]//MatrixForm=

1	1	−1	1	0	−29	2	−4
1	−1	0	1	0	5	0	4
1	3	−5	9	1	−243	10	−12
1	2	−3	4	0	−130	6	−8
1	−2	1	4	0	34	−2	8
1	−1	1	1	−1	41	−2	4
1	1	0	1	1	−17	0	−4
1	6	−14	36	5	−954	28	−24
1	3	−6	9	0	−351	12	−12
1	5	11	25	2	−697	−22	−20
1	4	8	16	0	−452	−16	−16
1	4	−8	16	2	−476	16	−16
1	1	−2	1	−1	−89	4	−4
1	−1	−1	1	1	17	2	4
1	−3	3	9	0	135	−6	12
1	0	1	0	−3	108	−2	0
1	−4	0	16	0	68	0	16
1	−3	1	9	−2	63	−2	12
1	−1	−3	1	−3	209	6	4
1	−2	3	4	−2	154	−6	8
1	2	2	4	3	−94	−4	−8
1	2	1	4	2	−58	−2	−8
1	−2	0	4	−3	46	0	8
1	3	3	9	3	−123	−6	−12
1	−1	2	1	−2	125	−4	4
1	−3	0	9	1	51	0	12
1	1	1	1	2	−53	−2	−4
1	0	0	0	−2	24	0	0
1	4	−7	16	3	−320	14	−16
1	1	−1	1	2	−53	2	−4
1	−1	0	1	−2	29	0	4
1	1	0	1	−1	7	0	−4
1	5	10	25	5	−565	−20	−20
1	2	−2	4	1	−70	4	−8
1	0	1	0	−1	84	−2	0

In[•]:=
```
NullSpace[mat]
```

Out[•]=
```
{{0, 4, 0, 0, 0, 0, 0, 1}, {0, 0, 2, 0, 0, 0, 1, 0}}
```

In[•]:=
```
MatrixForm@
 (mat = Table[{1, Vassiliev[2][K], Vassiliev[3][K], Vassiliev[2][K]^2, Vassiliev[4, 1][K],
    Vassiliev[4, 2][K], Together[∂_T Alexander[K][T]/(T - 1)] /. T → 1,
    Together[∂_{T,T} Alexander[K][T]] /. T → 1,
    Together[∂_{T,T,T} Alexander[K][T]] /. T → 1}, {K, AllKnots[{3, 8}]}])
```

Out[•]//MatrixForm=

1	1	−1	1	0	−29	2	2	−6
1	−1	0	1	0	5	−2	−2	6
1	3	−5	9	1	−243	6	6	−18
1	2	−3	4	0	−130	4	4	−12
1	−2	1	4	0	34	−4	−4	12
1	−1	1	1	−1	41	−2	−2	6
1	1	0	1	1	−17	2	2	−6
1	6	−14	36	5	−954	12	12	−36
1	3	−6	9	0	−351	6	6	−18
1	5	11	25	2	−697	10	10	−30
1	4	8	16	0	−452	8	8	−24
1	4	−8	16	2	−476	8	8	−24
1	1	−2	1	−1	−89	2	2	−6
1	−1	−1	1	1	17	−2	−2	6
1	−3	3	9	0	135	−6	−6	18
1	0	1	0	−3	108	0	0	0
1	−4	0	16	0	68	−8	−8	24
1	−3	1	9	−2	63	−6	−6	18
1	−1	−3	1	−3	209	−2	−2	6
1	−2	3	4	−2	154	−4	−4	12
1	2	2	4	3	−94	4	4	−12
1	2	1	4	2	−58	4	4	−12
1	−2	0	4	−3	46	−4	−4	12
1	3	3	9	3	−123	6	6	−18
1	−1	2	1	−2	125	−2	−2	6
1	−3	0	9	1	51	−6	−6	18
1	1	1	1	2	−53	2	2	−6
1	0	0	0	−2	24	0	0	0
1	4	−7	16	3	−320	8	8	−24
1	1	−1	1	2	−53	2	2	−6
1	−1	0	1	−2	29	−2	−2	6
1	1	0	1	−1	7	2	2	−6
1	5	10	25	5	−565	10	10	−30
1	2	−2	4	1	−70	4	4	−12
1	0	1	0	−1	84	0	0	0

In[•]:=
```
NullSpace[mat]
```

Out[•]=
```
{{0, 6, 0, 0, 0, 0, 0, 0, 1}, {0, -2, 0, 0, 0, 0, 0, 1, 0}, {0, -2, 0, 0, 0, 0, 1, 0, 0}}
```